# Efficient Communication Strategies For Distributed Signal Field Estimation


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***Abstract*— Wireless sensor networks are a promising architecture for monitoring large spatial areas. While recent years have seen a surge of research activity in sensor networks, many significant challenges need to be overcome to realize the vision of sensor networks. The key challenges are tied to two vital operations in a sensor network: efficient information routing between sensor nodes to extract useful information from the data collected by the sensors; and efficient communication of this information from the network to a certain destination. This paper proposes a novel collaborative communication and estimation scheme for distributed signal field estimation that overcomes these challenges by exploiting the underlying smoothness of the field. In our approach, the signal field is uniformly partitioned into multiple regions and the nodes in each region coherently communicate their measurements via a dedicated noisy multiple access channel (MAC) to the destination where the estimate of each region is constructed to give an estimate of the entire field. Two salient features of our scheme are: it requires relatively little collaboration among sensing nodes; and is potentially far more power-efficient due to the power pooling gain afforded by coherent cooperation of the nodes in each region. In this paper, we analyze our new approach under the simple setting of estimating a piece-wise constant field and show that optimal mean-square distortion scaling can be achieved at the destination with constant network power (vanishing per node power).**


## I. INTRODUCTION

Wireless sensor networks are an emerging technology that promise an unprecedented opportunity to monitor the remote physical world via wireless nodes that can sense the environment in various modalities, such as acoustic, seismic, infrared [1]–[3]. A wireless sensor network typically consists of a certain number of low cost sensor nodes equipped with small batteries that are deployed in a pre-defined or random manner inside the phenomenon of interest or very close to it. A wide variety of applications are being envisioned for sensor networks including disaster relief, border monitoring, contaminant tracking in the environment, and surveillance in battlefield scenarios.

An important problem in sensor networking applications is the estimation of spatially varying processes or fields. This could correspond to sensing the ambient temperature or humidity in a rainforest, sensing the intensity of oil contamination of ocean water in a certain area or sensing the intensity of a biochemical agent in the atmosphere. In this kind of an application, the goal of the sensor network is to sense the field, construct an estimate of the field, and communicate that estimate to a desired (typically remote) destination. This is a particularly challenging problem in sensor networks because of the strict limitations on the power consumption and processing power of the sensor nodes which, in turn, put constraints on the communication and estimation algorithms that can be employed inside the wireless sensor networks for field estimation. In this paper, we propose a novel collaborative communication and estimation scheme for distributed signal field estimation that combines existing results from wireless communications [4]–[7] and asymptotically gives us the same field estimate at the destination in terms of the mean-squared error as a centralized approach would give us while, at the same time, makes the sensor nodes expend minimal amount of power by employing optimal power allocation schemes in the sensor network. For the scope of this paper, we analyze our new approach under the simple setting of estimating a piece-wise constant field and provide metrics for quantifying the cost of communication and estimation, demonstrate the effect of different power allocation schemes on the accuracy of the final field estimate at the destination and show that optimal mean-square distortion scaling can be achieved at the destination with constant network power (vanishing per node power).

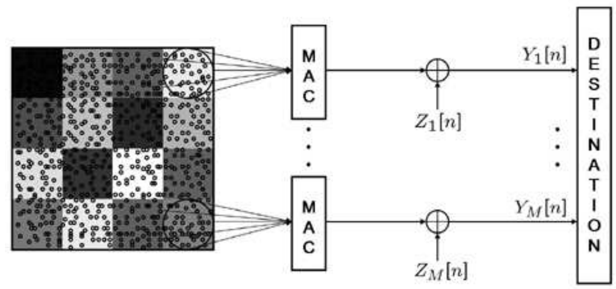


Fig. 1. A distributed architecture for estimating a piece-wise constant field with a wireless sensor network. The $N_r = N/M$ nodes in each of the $M$ regions communicate coherently to the destination via a dedicated MAC.


This research was supported in part by the NSF under grants CCR-0113385 and CCR-0310889, and by the ONR under grant N00014-00-1-0966.

### A. Problem Formulation

We assume that a two-dimensional spatial field, denoted by $F[n]$ ($n = 1, 2, \dots$), is being sensed over space and time by a wireless sensor network and that at any time instant $n$, the field is composed of $2^k$ equal sized *constant* regions, where $k \in \mathbb{N}$ (as shown in Fig. 1). Without loss of generality, the spatial domain of the field can be assumed to be the unit square, $[0, 1]^2$. The sensor network is assumed to be a collection of $N$ wireless nodes uniformly distributed on $[0, 1]^2$. Each node measures the field at its position which is contaminated with a zero-mean Gaussian noise sequence of variance $\sigma_W^2$ that is white in both the spatial and the temporal domain. This Gaussian noise corresponds to the measurement noise in the sensor network that could encompass ambient noise in the environment and electronic transducer noise sources. For the scope of this paper, we assume that local processing and computation at each sensor requires negligible amount of power and that sensors have a limited ability (in terms of the data rate) to noiselessly collaborate with each other.

The goal of the sensor network is to observe $F[n]$ over space and time and given some power constraint $P$ on the transmitted sensor signals, communicate an estimate $\widehat{F}[n]$ of the field to a destination which is assumed to be remote from the sensor network. The idea is to make $\widehat{F}[n]$ as close to $F[n]$ as possible, in terms of an appropriately chosen distortion measure $D$, while consuming minimal amount of power. The distortion measure that we are considering here is the mean-squared error (MSE) given by

$$D = E\left[\left|F - \widehat{F}\right|^2\right]. \tag{1}$$

In the absence of any noisy collaboration between the sensors (requiring additional power), the relevant metric for quantifying the cost of communication and estimation is the trade-off between the cost (power) $P$ of transmission and the achieved distortion level $D$ and the problem studied in this paper is that of finding a communication and estimation scheme that gives us the optimal trade-off between $P$ and $D$.

### B. Assumptions

We assume that each of the $2^k$ constant regions in the field can be modeled by a discrete memoryless zero mean Gaussian source $S_i[n]$ of variance $\sigma_S^2$, where $i = 1, 2, \dots, 2^k$ and $n = 1, 2, \dots$ Furthermore, $S_i[n]$ and $S_j[n]$ are assumed to be independent for all $i \neq j$ (implying that the field $F[n]$ has exactly $2^k$ degrees of freedom given by $2^k$ independent discrete memoryless zero mean Gaussian sources).

### C. Notation

Let $M = 2^k$ be the number of constant regions in the field and let $\mathcal{R}_i$ denote each region, where $i = 1, 2, \dots, M$. Let $N_i$ be the number of sensor nodes in the region $\mathcal{R}_i$. Because of the uniform distribution of sensor nodes in the field, we have

$$N_1 = N_2 = \cdots = N_M = N_r = \frac{N}{M}. \tag{2}$$

Let us denote the measurement of each sensor node in a particular region $\mathcal{R}_i$ and at a particular time instant $n$ by $X_{i,j}[n]$, where $i = 1, 2, \dots, M$, $j = 1, 2, \dots, N_r$ and $n = 1, 2, \dots$ Each sensor node belonging to a region $\mathcal{R}_i$ makes a measurement of the field at its location at a particular time instant $n$, corresponding to observing $S_i[n]$, which is contaminated with a zero-mean white Gaussian noise sequence $W_{i,j}[n]$ of variance $\sigma_W^2$. As already mentioned, $W_{i,j}[n]$ is assumed to be white in both the spatial and the temporal domain implying $W_{i,j}[n]$ and $W_{k,l}[m]$ are independent for all $i \neq k$, $j \neq l$ and $n \neq m$. Thus, $X_{i,j} \sim \mathcal{N}\left(0, \sigma_S^2 + \sigma_W^2\right)$ and is given by

$$X_{i,j}[n] = S_i[n] + W_{i,j}[n]. \tag{3}$$

Let $T_{i,j}[n]$ be the signal transmitted by each of the sensor nodes in the region $\mathcal{R}_i$ at time $n$. The constraint on the signals transmitted by the sensors in the network is a sum power constraint given by

$$\sum_{i=1}^{M} \sum_{j=1}^{N_r} E\left[|T_{i,j}|^2\right] = P. \tag{4}$$

Because of the homogeneity of $F[n]$, the sum power constraint given in (4) can be further broken down into a sum power constraint on the signals transmitted by the sensors in any region $\mathcal{R}_i$ and a power constraint on the signal transmitted by any sensor. That is,

$$\sum_{j=1}^{N_r} E\left[|T_{i,j}|^2\right] = P_r \tag{5}$$

and

$$E\left[|T_{i,j}|^2\right] = P_n. \tag{6}$$

### D. Related Work

First of all, let us comment on the field model used in our approach. We assume that the field $F[n]$ is a piece-wise constant field having $M$ independent degrees of freedom. Although far from a realistic scenario, this setting really helps in understanding the basics of our scheme and clearly demonstrates the effects of different power allocation schemes on the accuracy of the final field estimate at the destination. We refer the reader to [8] for an extension of this work under a more general and realistic field setting.

Secondly, most previous work in this area has focused on multihop communication schemes and "in-network" data processing and compression [7], [9]–[11]. This requires a significant level of network infrastructure, and the theoretical approaches in the works above generally assume this infrastructure as given. Our new approach, in contrast to previous methods, eliminates the need for in-network communications and processing and instead requires only local synchronization among nodes and is, therefore, potentially far more power-efficient.

## II. Optimal Centralized Estimation

We first consider the structure of an optimal centralized estimator in which all the sensor measurements $\{X_{i,j}[n]\}$ are available noise free at the destination. The distortion scaling of this centralized estimator serves as a benchmark for assessing the performance of any distributed scheme. Let us denote this centralized benchmark distortion by $D_{cent}$. We say that the distortion $D$ of a field estimate achieves the benchmark distortion if, for some large enough $N$, we get $D = CD_{cent}$, where $C$ is some constant and we write it as $D \sim D_{cent}$.

Now, since $MSE = bias^2 + variance$, optimal centralized distortion scaling for a piece-wise constant field can be achieved by using the least squares fit of a constant to the data in each constant region of the field (resulting in $bias^2 = 0$ and $MSE = variance$). Let $\widehat{S}_{i,cent}[n]$ be the centralized estimate of $S_i[n]$. Since the field is being estimated by using constant fits (averaging) on each region of the field, we have

$$\widehat{S}_{i,cent}[n] = \frac{1}{N_r}\sum_{j=1}^{N_r} X_{i,j}[n] \tag{7}$$

and the resulting field estimate is given by

$$\widehat{F}_{cent}(x,y)[n] = \sum_{i=1}^{M} \widehat{S}_{i,cent}[n]\ 1_{\{(x,y)\in\mathcal{R}_i\}} \tag{8}$$

where $(x,y) \in [0,1]^2$ and $1_{\{(x,y)\in\mathcal{R}_i\}} = 1$ if $(x,y)$ is in the region $\mathcal{R}_i$ and zero otherwise. The distortion of the centralized field estimate $\widehat{F}_{cent}[n]$ can then be given by

$$\begin{aligned} D_{cent} &= E\left[\left|F - \widehat{F}_{cent}\right|^2\right] = \sum_{i=1}^{M} E\left[\left|S_i - \widehat{S}_{i,cent}\right|^2\right] \\ &= \sum_{i=1}^{M}\frac{\sigma_W^2}{N_r} = \frac{M^2\sigma_W^2}{N} = \frac{2^{2k}\sigma_W^2}{N} = \frac{C}{N}, \end{aligned} \tag{9}$$

where $C = 2^{2k}\sigma_W^2$ is a constant independent of $N$. Therefore, from (9), the benchmark distortion $D_{cent}$ is given by[1]

$$D_{cent} \sim \mathcal{O}\left(\frac{1}{N}\right). \tag{10}$$

## III. Collaborative Communication and Estimation Scheme

We now analyze the distortion of the field estimate when the estimation process must be carried out in conjunction with data communication over noisy channels. That is, the $N_r$ sensor nodes in each region $\mathcal{R}_i$ of the field must transmit the constant fit at time $n$ of the region (corresponding to $S_i[n]$) over a noisy communication channel to the destination using some coding strategy (as shown in Fig. 1). Let the communication channel be a multiple-access (MAC) additive white Gaussian noise channel, where the communication noise is given by the sequence $Z_i[n] \sim \mathcal{N}(0,\sigma_Z^2)$ $(i = 1,2,\ldots,M)$.[2] Also, let $Z_i[n]$ and $Z_j[n]$ be independent for all $i \neq j$. Let $Y_i[n]$ be the signal received by the destination from the region $\mathcal{R}_i$, then

$$Y_i[n] = \sum_{j=1}^{N_r} T_{i,j}[n] + Z_i[n] \tag{11}$$

and the goal of the destination is to construct an estimate $\widehat{F}[n]$ of the field having distortion $D$ as close to the benchmark distortion $D_{cent}$ as possible. In order to achieve this goal, the destination must construct estimates $(\widehat{S}_i[n])$ of $S_i[n]$ with minimum distortion using the outputs $Y_i[n]$ of MAC channels. Let $D_r$ denote the distortion of each $\widehat{S}_i[n]$ (region distortion). Then, the total distortion $D$ of $\widehat{F}[n]$ is given by

$$D = E\left[\left|F - \widehat{F}\right|^2\right] = \sum_{i=1}^{M} E\left[\left|S_i - \widehat{S}_i\right|^2\right] = MD_r \tag{12}$$

and hence, minimization of $D$ to make it closer to $D_{cent}$ would require minimization of $D_r$ . Therefore, we now need a coding strategy at the sensors in each region $\mathcal{R}_i$ and a decoding strategy at the destination that gives us the lowest (possible) distortion $D_r$ in $\widehat{S}_i[n]$ for a given region power constraint $P_r$ or in other words, gives us the optimal trade-off between $D_r$ and $P_r$. However, considering each region $\mathcal{R}_i$ as a sensor network of $N_r$ sensor nodes in itself, it has been shown in [4] that for such a class of Gaussian sensor networks the optimal trade-off between $D_r$ and $P_r$ can be obtained by employing uncoded transmission at the sensor nodes and a minimum MSE (MMSE) estimator at the destination. Therefore, for the optimal trade-off between $D_r$ and $P_r$, the transmitted signal at the sensors must be of the form

$$T_{i,j}[n] = \sqrt{\frac{P_n}{\sigma_S^2 + \sigma_W^2}}X_{i,j}[n] \tag{13}$$

and the region estimator at the destination must be of the form

$$\widehat{S}_i[n] = \frac{\sqrt{\frac{P_n}{\sigma_S^2+\sigma_W^2}}N_r\sigma_S^2}{\frac{N_rP_n}{\sigma_S^2+\sigma_W^2}\left(N_r\sigma_S^2+\sigma_W^2\right)+\sigma_Z^2}Y_i[n]. \tag{14}$$

Using (11), (13) and (14), the resulting region distortion for each region $\mathcal{R}_i$ is given by

$$D_r = \frac{\sigma_S^2\sigma_W^2}{\frac{N_r^2}{N_r+\left(\sigma_Z^2/\sigma_W^2\right)\left(\sigma_S^2+\sigma_W^2\right)/P_n}\sigma_S^2+\sigma_W^2} \tag{15}$$

and using (12) and (15), the total distortion of the field estimate $\widehat{F}[n]$ is given by

$$D = \frac{M\sigma_S^2\sigma_W^2}{\frac{N_r^2}{N_r+\left(\sigma_Z^2/\sigma_W^2\right)\left(\sigma_S^2+\sigma_W^2\right)/P_n}\sigma_S^2+\sigma_W^2}. \tag{16}$$

[1]Given some function $f(\alpha)$, we say $f$ is '*of the order of*' $\alpha$ if, for large enough $\alpha$, we have $f = C\alpha$, where $C$ is some constant and we write it as $f \sim \mathcal{O}(\alpha)$.

[2]We are implicity assuming that separate (independent) MAC channels can be readily setup from each region to the destination.

### A. Asymptotic Optimality of the Scheme

If we assume that all the MAC channels between the $M$ regions and the destination are noise free ($\sigma_Z^2 = 0$) then, from (2) and (16), the distortion measure of $\widehat{F}[n]$ at the destination is given by

$$D = \frac{M^2 \sigma_W^2}{N + M \frac{\sigma_W^2}{\sigma_S^2}} \tag{17}$$

and since $M$ is not a function of $N$, we get

$$D \sim \mathcal{O}\left(\frac{1}{N}\right) \tag{18}$$

for $N \gg M\sigma_W^2/\sigma_S^2$. Hence, in the absence of communication noise, our proposed scheme is asymptotically optimal since $D \sim D_{cent}$. We now show that even with noisy communication links ($\sigma_Z^2 \neq 0$), our proposed scheme achieves the benchmark distortion in the limit of large number of sensors.

In order to show this, let us consider the case where the power constraint $P_n$ on the signal transmitted by each sensor remains fixed as the number of sensors in the network increases ($P_n(N) = P_{n_o}$). Then, from (2) and (16), the distortion $D$ is given by

$$D = \frac{M^2 \sigma_W^2}{\frac{N^2}{N + M\left(\sigma_Z^2/\sigma_W^2\right)\left(\sigma_S^2 + \sigma_W^2\right)/P_{n_o}} + M\frac{\sigma_W^2}{\sigma_S^2}} \tag{19}$$

and since $P_{n_o}$ is a constant and $M$ is not a function of $N$, we again have

$$D \sim \mathcal{O}\left(\frac{1}{N}\right) \sim D_{cent} \tag{20}$$

for $N \gg \max\left(M\left(\sigma_Z^2/\sigma_W^2\right)\left(\sigma_S^2 + \sigma_W^2\right)/P_{n_o}, M\sigma_W^2/\sigma_S^2\right)$.

## IV. Power-Distortion Scaling Laws

In this section, we look at how different power allocation schemes affect the distortion of the final field estimate at the destination and which power allocation scheme gives us the best trade-off between $P$ and $D$. We have seen in the last section that even in the presence of communication noise, we get $D \sim D_{cent}$ (as $N$ gets large enough) provided that all the sensor nodes in the network transmit with some fixed power that is independent of $N$ ($P_n(N) = P_{n_o}$). From (4) and (5), however, we see that this power allocation scheme heavily burdens the power resources of the network since it results in $P(N) \sim \mathcal{O}(N)$ and $P_r(N) \sim \mathcal{O}(N)$. This brings us to the question of *Can we do better?* That is, can we find a more stringent power allocation scheme which results in $P(N) \sim \mathcal{O}\left(N^\beta\right)$, where $\beta < 1$ and still give us $D \sim D_{cent}$?

Finding answer to the above question requires careful analysis of (16). As can be seen in (16), the distortion in the final field estimate $\widehat{F}[n]$ at the destination is a result of the measurement noise $\{W_{i,j}[n]\}$ and the communication noise $\{Z_i[n]\}$. The effect of measurement noise on distortion can only be reduced by increasing the spatial density of the sensor nodes in the network and the effect of communication noise on distortion can only be reduced by putting more power into the signals transmitted by the sensor nodes. Thus, given a certain

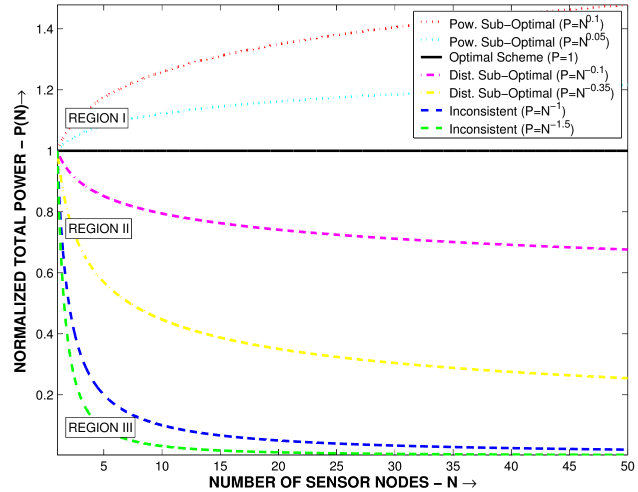


Fig. 2. Illustration of the different scaling regions for the total network power, $P(N)$.

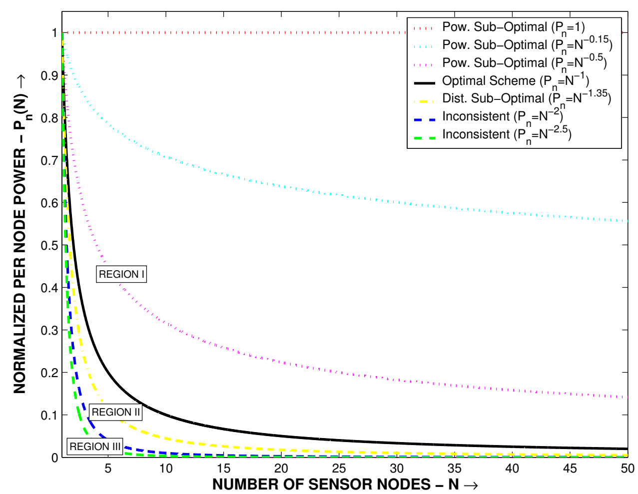


Fig. 3. Illustration of the different scaling regions for the per node power, $P_n(N)$.

number of sensors $N$ in the network, the only way to reduce the distortion is by putting more power into each transmitted signal. This, however, holds true up to a certain critical optimal power per node and beyond that point, putting more power in the network would have no impact on distortion because of the limiting factor of measurement noise which is unaffected by the power allocation. Therefore, the optimal power allocation scheme corresponds to allocating just enough power in the network so that the effect of the measurement noise is balanced by the effect of the communication noise and this leads us to the following power-distortion scaling laws (see Fig. 2 and Fig. 3).

*Optimal Power Allocation Scheme:* The optimal power allocation scheme corresponds to allocating a fixed (constant) power to the entire sensor network regardless of $N$ ($P(N) = P_{opt} = P_o$). From (4) and (5), this results in $P_r(N) = P_o/2^k$ (a constant) and $P_n(N) \sim \mathcal{O}\left(\frac{1}{N}\right)$. In this case, not only does the distortion $D$ achieve the benchmark distortion $D_{cent}$ but also this power allocation scheme leads to the sensor network consuming minimal amount of power when compared with any other power allocation scheme that gives us $D \sim D_{cent}$.

*Measurement-Limited Region:* Increasing $P$ at any rate as $N$ increases ($P(N) \sim \mathcal{O}\left(N^{\beta}\right)$ for $\beta > 0$) has no impact on $D$ and we still get $D \sim D_{cent}$. This power allocation scheme is clearly power sub-optimal since it makes the sensor network consume more power than needed to achieve $D \sim D_{cent}$. In terms of $P_r$ and $P_n$, this power allocation scheme can be given as $P_r(N) \sim \mathcal{O}\left(N^{\beta}\right)$ and $P_n(N) \sim \mathcal{O}\left(N^{(\beta-1)}\right)$ for $\beta > 0$. This power allocation scheme is shown as REGION I in Fig. 2 and Fig. 3.

*Communication-Limited Region:* If we are willing to loosen our requirement of $D \sim D_{cent}$, then we can decrease $P$ at a certain rate as $N$ increases and still asymptotically drive the distortion $D$ to 0, though at a comparatively slower rate since, in this case, the distortion scaling is limited by the communication noise. In order to operate in this distortion sub-optimal region, we must have $P(N) \sim \mathcal{O}\left(N^{\beta}\right)$ for $-1 < \beta < 0$ and this results in $D \sim \mathcal{O}\left(\frac{1}{N^{(1+\beta)}}\right)$, $P_r(N) \sim \mathcal{O}\left(N^{\beta}\right)$ and $P_n(N) \sim \mathcal{O}\left(N^{(\beta-1)}\right)$. This power allocation scheme is shown as REGION II in Fig. 2 and Fig. 3.

*Inconsistent Region:* The distortion $D$ cannot be asymptotically driven down to 0 if the rate at which $P$ decreases with increasing $N$ is too fast. This corresponds to $P(N) \sim \mathcal{O}\left(N^{\beta}\right)$, $P_r(N) \sim \mathcal{O}\left(N^{\beta}\right)$ and $P_n(N) \sim \mathcal{O}\left(N^{(\beta-1)}\right)$ for $\beta \leq -1$. This (inconsistent) power allocation scheme is shown as REGION III in Fig. 2 and Fig. 3.

## V. Conclusions and Future Work

In this paper, we have proposed a novel collaborative communication and estimation scheme for distributed signal field estimation that asymptotically gives us the same field estimate at the destination in terms of the mean-squared error as a centralized approach would give us while, at the same time, makes the sensor nodes expend minimal amount of power by employing optimal power allocation schemes in the sensor network.

For the scope of this paper, we have analyzed our new approach under the simple setting of estimating a piece-wise constant field, provided metrics for quantifying the cost of communication and estimation and demonstrated the effect of different power allocation schemes on the accuracy of the final field estimate at the destination. To summarize, we have shown that optimal mean-square distortion scaling ($D \sim D_{cent}$) can be achieved at the destination with constant network power (vanishing per node power) and the distortion of the final field estimate at the destination can be driven to zero asymptotically as long as the per node power $P_n(N)$ decays just a little slower than $1/N^2$. This is remarkable since it says that consistent field estimation is possible in the limit of a large number of sensor nodes even if the total network power $P(N)$ goes to zero! Our future work includes extensions to fields with discontinuities.

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